Chapter 5

Heterogenous firms

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Road map

1. Introduction: Firms in international trade: stylized facts
2. The Melitz (2003) model
3. Extensions of the Melitz model

1. Introduction

New trade theories are twofold:

- Oligopolistic models ("reciprocal dumping")
  - Pro: the gains of trade
    - Pro-competitive effect (Trade openness exerts a pressure on firms' margins and prices)
    - Rationalization effect (foreign competitors wipe out the least efficient firms)
  - Cons: partial equilibrium and rapid growth of complexity

- Monopolistic competition models ("DSK")
  - Pro:
    - Highly tractable general equilibrium model
      - » Possible to mix up several market structures, to study distributional effects, to insert an engine of (endogenous) growth, etc.
    - Very convincing motive for intra-industry trade
      - » Demand for variety
    - Obvious empirical support from gravity equations
  - Cons:
    - No pro-competitive and rationalization effects

1. Introduction

Pro-competitive effect is clearly something we expect to see in a comprehensive trade theory

More, DSK framework assumes that all firms are the same: all of them export and have the same performances abroad…

- At odds with empirical evidence: Exporting is for a few
- At odds with policy-oriented issues related to firms' competitiveness

1. Introduction

Early 2000:

- Melitz (2003), Melitz & Ottaviano (2008), Chaney (2008)…
- Monopolistic competitive models with heterogeneous firms
  - General equilibrium
  - Pro-competitive effects (at least rationalization effect)
  - (Pseudo-) missing link between Brander-Krugman and Krugman.
  - = “New new” trade theory

1. Firms in international trade

Repeated evidence on the difficulty faced by firms aiming to enter foreign markets

- Data for: USA, France, Ireland, Columbia, Mexico, China…
- Export is definitively for the few… and the best:
  - Only a small subset of firm export
  - Among exporters, only a few export to many countries
  - Exporter are:
    - Bigger, more productive, pay higher wage…

1. Firms in international trade: exporting is for the few

Bernard, Eaton, Jensen and Kortum

US plant data for 1992

Only 18% of plants export something

Average exporter sells only 14% of its output

Table 2

Exporting By U.S. Manufacturing Firms, 2002

NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	9	15
Aggregate manufacturing	**100**	$\boxed{\textbf{18}}$	$\boxed{\textbf{14}}$

Sources: Data are from the 2002 U.S. Census of Manufactures.
Notes: The first column of numbers summarizes the distribution of manufacturing firms across three-digit NAICS manufacturing industries. The second reports the share of firms in each industry that export. The final column reports mean exports as a percent of total shipments across all firms that

1. Firms in international trade: exporting is for the few

Mayer & Ottaviano ("Happy few" report)

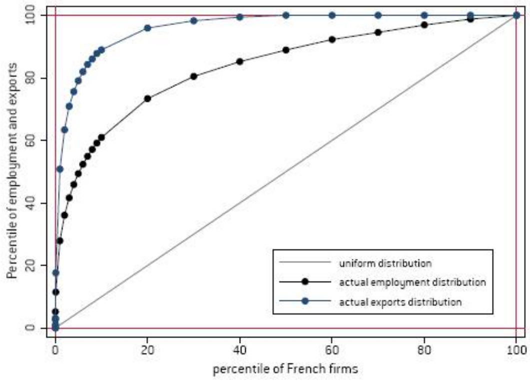
Evidence from several EU countries

- Exporters are the few
- Most exporters are "small ones"
- Most countries' exports are driven by a small fraction of big exporters

1. Firms in international trade: exporting is for the few

The concentration of exports is much stronger than employment

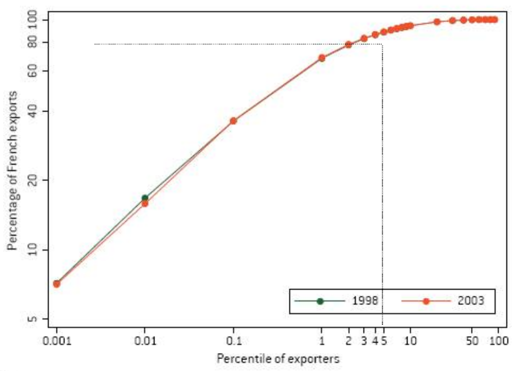
Figure 1: The superstar exporters phenomenon (France, restricted sample)



1. Firms in international trade: exporting is for the few

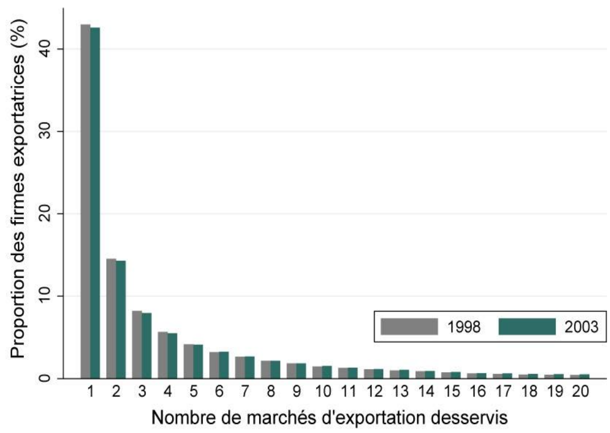
Top 1% of French exporters make 68% of exports

Figure 2: The superstar exporters phenomenon, logarithmic transformation (France, exhaustive sample)



1. Firms in international trade: exporting is for the few

Most exporters export to a very small numbers of countries (France)



1. Firms in international trade: exporting is for the few

Firms exporting to more than 10 countries account for 85.44% of total exports

Table 3: Distribution of French exporters over products and markets[9]

Share of French exporters in 2003 (total number exporters: 99259)

	Number of countries			
No. of products	1	5	10+	Total
1	29.61	0.36	0.22	34.98
5	0.76	0.45	0.62	4.73
10+	0.95	0.89	10.72	18.57
Total	42.59	4.12	15.54	100

Share of French exports in 2003 (total exports: 314.3 billion €)

	Number of countries			
No. of products	1	5	10+	Total
1	0.7	0.08	0.38	1.86
5	0.3	0.08	1.06	1.97
10+	0.28	0.45	76.3	81.36
Total	2.85	1.55	85.44	100

Source: EFIM.

1. Firms in international trade: exporting is for the best

Exporters are bigger,
pay higher wage,
are more capital
Intensive

(Source: BEJK)

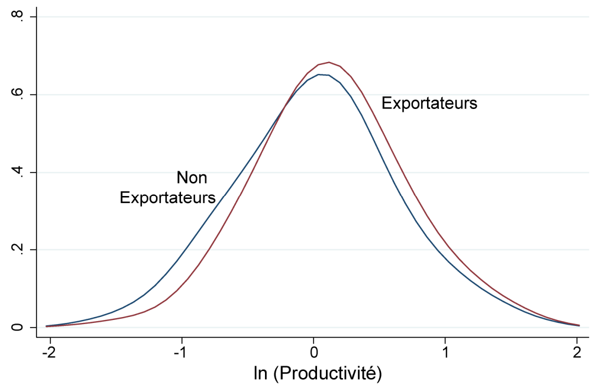
Table 3
Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	(2)	(3)
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

1. Firms in international trade: exporting is for the best

Exporters are more productive (French Firms)



1. Firms in international trade: Trade margins

Total trade can be decomposed into the # of shipments, the mean quantity exported and the mean price

Table 6

Gravity and Aggregate U.S. Exports, 2000

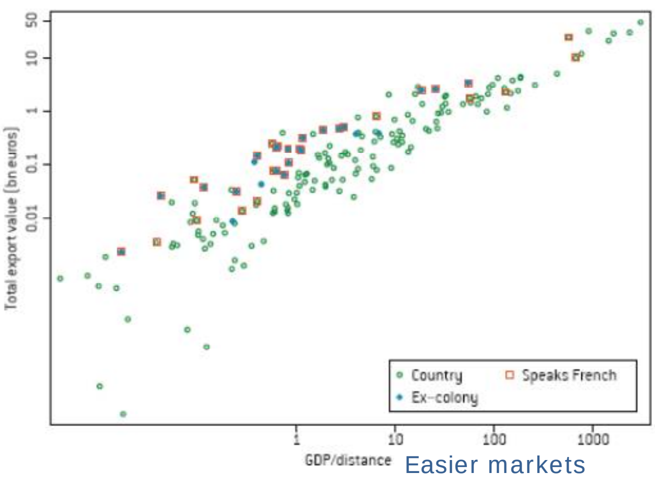
Extensive margins — Intensive margin

	Log of total exports value	*Log of number of exporting firms*	*Log of number of exported products*	*Log of export value per product per firm*
Log of GDP	0.98	0.71	0.52	−0.25
	(0.04)	(0.04)	(0.03)	(0.04)
Log of distance	−1.36	−1.14	−1.06	0.84
	(0.17)	(0.16)	(0.15)	(0.19)
Observations	175	175	175	175
R^2	0.82	0.74	0.64	0.25

Sources: Data are from the 2000 Linked-Longitudinal Firm Trade Transaction Database (LFTTD).
Notes: Each column reports the results of a country-level ordinary least squares regression of the dependent variable noted at the top of each column on the covariates noted in the first column. Results for the constant are suppressed. Standard errors are noted below each coefficient. Products are defined as ten-digit Harmonized System categories. All results are statistically significant at the 1 percent level.

1. Firms in international trade: Trade margins

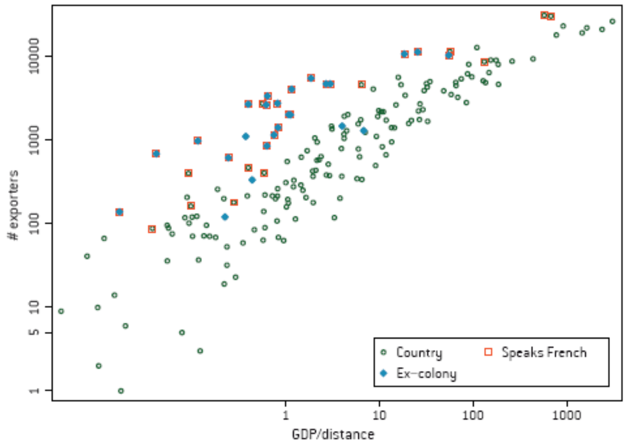
Figure 16: The forces of gravity for France in 2003



Easier markets

1. Firms in international trade: Trade margins

Figure 17: The extensive margin



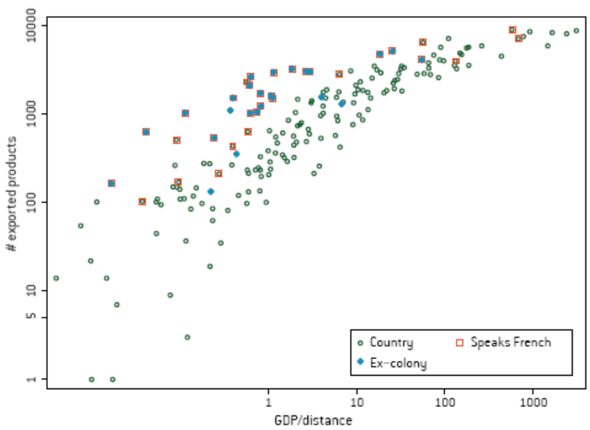
(a) gravity for # of firms

It is easier to export to large and proximate markets…

… and to those sharing some cultural and historical links

1. Firms in international trade: Trade margins

Extensive margin (products)



(b) gravity for # of products

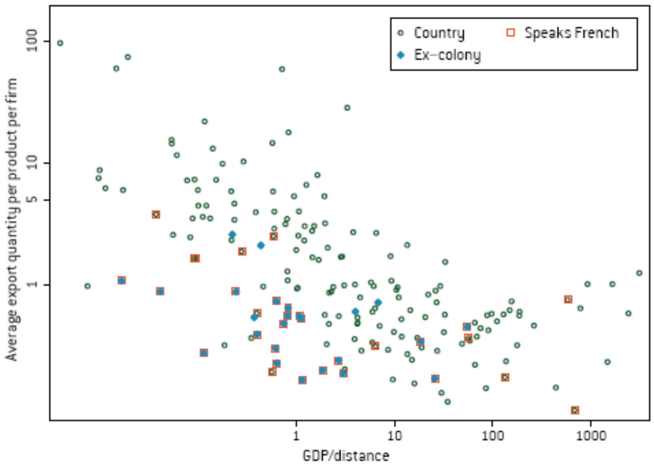
It is easier to export **more products** to large and proximate markets…

… and to those sharing some cultural and historical links

EFIM

1. Firms in international trade: Trade margins

Figure 18: The intensive margin

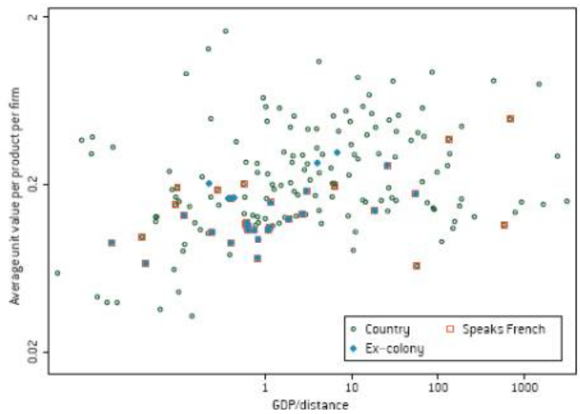


On easy market, the mean exported value is smaller:

It is probably a composition effect (more small firms in easy markets)

1. Firms in international trade: Trade margins

Intensive margin (price)



(b) gravity for average price

: EFIM

Mean price is (slightly) larger on easy markets

1. Firms in international trade

These findings suggest that the most productive firms self-select into export markets

The extensive margin of trade (i.e., the number of firms selling in each market) plays a significant role in explaining trade flows.

Standard DSK model (or HK models) does not feature these facts

2. The Melitz model: closed economy

These slides follow Melitz and Redding (2012)

The economy consists of *J+1* sector and consumers have the following preferences:

$$U = \sum_{j=0}^{J} \beta_j \log Q_j, \qquad \sum_{j=0}^{J} \beta_j = 1, \ \beta_j \geq 0.$$

"Outside good": Industry *j = 0* (CRS-PC, numeraire good, produced in all countries).

The *J* other industries produce differentiated products and preferences are CES

$$Q_j = \left[\int_{\omega \in \Omega_j} q_j(\omega)^{(\sigma_j - 1)/\sigma_j} d\omega \right]^{\sigma_j/(\sigma_j - 1)}, \qquad \sigma_j > 1, \ j \geq 1.$$

2. The Melitz model: basic framework

Each industry receives a constant share of the aggregate income.

Total demand addressed to industry *j* is: $X_j = \beta_j Y$

And the demand for each individual variety ω is:

$$q_j(\omega) = A_j p_j(\omega)^{-\sigma_j}, \qquad A_j = X_j P_j^{\sigma_j - 1},$$

With: $P_j = \left[\int_{\omega \in \Omega_j} p(\omega)^{1-\sigma_j}\right]^{1/(1-\sigma_j)}$

2. The Melitz model: closed economy

Technology

- Varieties in industry *j* are produced with a single factor (or a composite factor), *L* with wage *w*.
- Production of each variety involves a fixed production cost and a constant marginal cost, wich is inversely proportional to firms' productivity φ.
- To produce the quantity q_j a given firm must employ:

$$l_j = f_j + \frac{q_j}{\varphi}$$

2. The Melitz model: closed economy

Technology

(dropping *j* subscript)

– Dixit Stiglitz monopolistic competition (with CES) leads to constant mark-ups:

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$

– Then, firm's revenue is:

$$r(\varphi) = Ap(\varphi)^{1-\sigma} = A\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\varphi^{\sigma-1}$$

– And firm's profit is:

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf = B\varphi^{\sigma-1} - wf, \qquad B = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}w^{1-\sigma}A.$$

2. The Melitz model: closed economy

Firm performance measures and productivity

– A key implication of the CES is that the relative outputs and revenues of firms depends solely on their relative productivities:

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}, \qquad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}, \qquad \varphi_1, \varphi_2 > 0.$$

(logically higher elasticity of substitution implies greater difference in firm size for a given difference in productivity)

2. The Melitz model: closed economy

Zero profit condition:

- There is a competitive fringe of potential entrants.
- They can enter by paying a sunk cost of f_E units of labor in order to draw their productivity φ from a distribution g(φ), with cumulative $G(\varphi)$.
- After observing its productivity, a firm decides whether to exit or produce.
- All firms with a productivity $\varphi > \varphi^*$ will produce
- The firm with the threshold productivity φ^* is indifferent between producing or not:

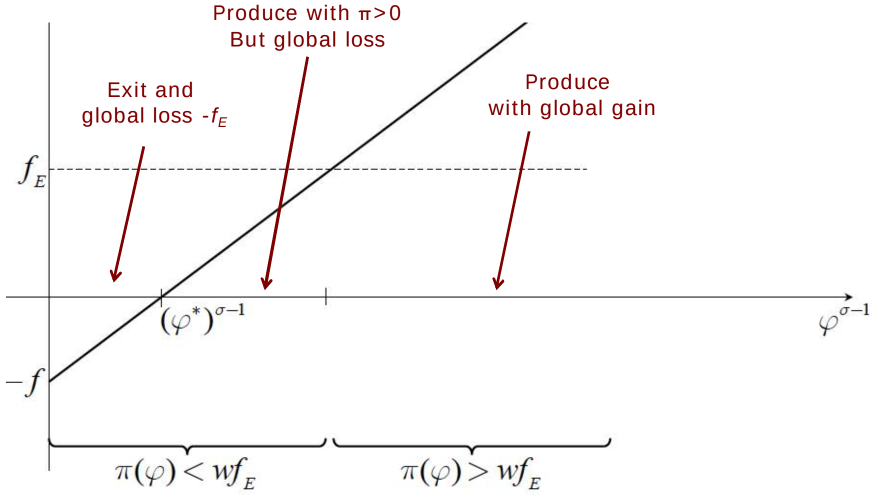
$$\pi(\varphi^*) = \frac{r(\varphi^*)}{\sigma} - wf = \boxed{B(\varphi^*)^{\sigma-1} - wf = 0}$$

2. The Melitz model

We have three groups of firms:

- **Unlucky firms** which draw a productivity $\varphi < \varphi^*$ exit immediately the market, before starting the production. They receive $\pi(\varphi)=0$… hence making a loss once the sunk cost is taken into account.
- **very lucky firms**. All Firms with $\varphi > \varphi^*$ receive $\pi(\varphi)>0$. But some of them have a productivity such that $\pi(\varphi)>wf_E$, so that they make positive profits net of the sunk cost of entry.
- **Less lucky firms**, have $\pi(\varphi)>0$ and enter the market, but have a productivity such that $\pi(\varphi)<wf_E$.

2. The Melitz model



2. The Melitz model

Free entry condition:

Given these 3 groups of firms, free entry implies that, in equilibrium, the ***expected measure of ex-ante*** profits (inclusive of the entry cost) is zero:

$$\int_0^\infty \pi(\varphi)dG(\varphi) = \int_{\varphi^*}^\infty \left[B\varphi^{\sigma-1} - wf\right] dG(\varphi) = wf_E$$

Where *G(φ)* is the cumulative distribution of productivities

2. The Melitz model

Closed economy sectoral equilibrium

Sectoral equilibrium is defined by the following variables (specific to each sector *j* but we omit the subscript *j* here):

- The productivity cutoff: φ^*
- The wage *w* (not specific to *j*)
- The labor force engaged in the industry, *L*

Zero-profit and Free-entry conditions provide two equations involving only two endogenous variables: the cutoff φ^* and the index of market demand B/w.

2. The Melitz model: closed economy

Let's combine the ZPC and the FE:

ZPC gives:

$$B(\varphi^*)^{\sigma-1} = wf \Rightarrow B = wf\,/\,(\varphi^*)^{\sigma-1}$$

Then, with FE, we have:

$$\int_{\varphi^*}^{\infty}\left[wf\,\frac{\varphi^{\sigma-1}}{(\varphi^*)^{\sigma-1}} - wf\right]dG(\varphi) = wf_E$$

$$\Rightarrow fJ(\varphi^*) = f_E \quad \text{with} \quad J(\varphi^*) = \int_{\varphi^*}^{\infty}\left[\frac{\varphi^{\sigma-1}}{(\varphi^*)^{\sigma-1}} - 1\right]dG(\varphi)$$

2. The Melitz model: closed economy

$$J(\varphi^*) = f_E / f > 0 \quad \text{with} \quad J(\varphi^*) = \int_{\varphi^*}^{\infty} \left[\frac{(\varphi)^{\sigma-1}}{(\varphi^*)^{\sigma-1}} - 1 \right] dG(\varphi)$$

J is monotonicaly decreasing with φ*:

$$\lim_{\varphi^* \to 0} J(\varphi^*) = \infty \quad \text{and} \quad \lim_{\varphi^* \to \infty} J(\varphi^*) = 0$$

… which identifies a unique φ* such that $J(\varphi^*) = f_E / f > 0$

2. The Melitz model: closed economy

Some intuitions about ZPC and FE:

- As market size increases, firms make more profits and firms with lower productivity can enter.

➔ ZPC involves a negative relationship between B/w and φ*

- The FE condition limits the number of entries by imposing that the sum of profits in the economy is constant.
- In a bigger market, firms make more profits
- To keep the sum of profits unchanged, less firms must enter.

➔ FE involves a positive relationship between B/w and φ*

- The two functions relating B/w and φ* have opposite slope ➔ equilibrium is unique

2. The Melitz model: closed economy

Sectoral labor market clearing:

- The mass M of active firms is the share of mass M_E of entrants that are successful and survive.
- This share depends on the productivity cutoff such that:

$$M = [1 - G(\varphi^*)] M_E$$

- The sectoral labor supply serves the production of all active firms and covers the sunk cost for all entrants. In addition, the payment to production workers is the residual between aggregate sector revenue and aggregate profits. Then:

$$L = \frac{R - \Pi}{w} + M_E f_E$$

2. The Melitz model: closed economy

Sectoral labor market clearing:

$$L = \frac{R - \Pi}{w} + M_E f_E$$

- Free entry condition ensures that aggregate profits, Π, exactly covers the aggregate entry cost, $wM_E f_E$.
- Then $R/w=L$.
- In a closed economy, the revenue is equal to the sectror's expenditures. We have:

$$L = R/w = X/w$$

Note: Here, market size does not affect the cutoff value nor average firms' sales (just like in Krugman (1980) where firm size is independent of market size) on a larger market, the mass of firms will be larger, leaving unchanged the average size of firms. This is very specific to the CES assumption.

2. The Melitz model: closed economy

General equilibrium (reintroducing all the industries)

Assuming that labor moves freely across industries, wage are equalized.

Assuming an outside good (or choosing labor as a numéraire):

$$w_j = w = 1$$

Sectoral market size is given by product shares (β) and total workforce:

$$R_j = X_j = \beta_j Y = \beta_j w \bar{L}.$$

2. The Melitz model: closed economy

General equilibrium

Let's use the formula for the expected profit (free entry condition):

$$\int_0^\infty \pi(\varphi) dG(\varphi) = \int_{\varphi^*}^\infty \left[B\varphi^{\sigma-1} - wf \right] dG(\varphi) = wf_E$$

This expected profit is equal to $\left[1 - G(\varphi^*)\right]\bar{\pi}$, where $\bar{\pi}$ is the average profit.

Then, $\frac{\bar{\pi}}{w} = \frac{f_E}{1 - G(\varphi^*)}$

Using $\frac{\bar{r}}{w} = \sigma\left(\frac{\bar{\pi}}{w} + f\right)$

… the mass of firm is thus $M_j = \frac{R_j}{\bar{r}_j} = \frac{\beta_j \bar{L}}{\sigma\left[\frac{f_E}{1-G(\varphi^*)} + f\right]}$

2. The Melitz model: open economy

Open economy

N countries (*i=1..N*) with same preferences and technologies.

The (freely traded) outside good pins down the wage

Transport costs to serve country *n*:

- Iceberg variable transport cost τ_{ni}
- Fixed cost of exporting (advertising, creation of a distribution network, conforming to foreign regulations…), f_{ni}

CIF price on market *n*: $p_{ni}(\varphi) = \frac{\sigma}{\sigma - 1}\frac{\tau_{ni}}{\varphi}$

2. The Melitz model: open economy

Open economy

Revenue and profit on that destination:

$$r_{ni}(\varphi) = A_n p_{ni}(\varphi)^{1-\sigma}, \qquad A_n = X_n P_n^{\sigma-1},$$

$$\pi_{ni}(\varphi) = B_n \tau_{ni}^{1-\sigma} \varphi^{\sigma-1} - f_{ni}, \qquad B_n = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} A_n.$$

2. The Melitz model: open economy

Fixed market access cost implies that there is a cut-off productivity level below which firms make negative profit on market *n:*

$$\pi_{ni}(\varphi^*_{ni}) = 0,$$

$$\frac{r_{ni}(\varphi^*_{ni})}{\sigma} = f_{ni} \qquad \Longleftrightarrow \qquad B_n (\tau_{ni})^{1-\sigma} (\varphi^*_{ni})^{\sigma-1} = f_{ni},$$

Firms with $\varphi < \varphi^*_{ni}$ do not export to country *n* and receive zero profit on this market

2. The Melitz model: open economy

- Just like in the closed economy, the free entry condition for country *i* equates entrants' ex-ante expected profit with the sunk cost.
- It is now:

$$\int_0^\infty \pi_i(\varphi) dG_i(\varphi) = \sum_n \int_{\varphi^*_{ni}}^\infty \left[B_n \tau_{ni}^{1-\sigma} \varphi^{\sigma-1} - f_{ni}\right] dG_i(\varphi) = f_{Ei}.$$

- Using the definition for *J* given above, this condition can be rewrote:

$$\sum_n f_{ni} J_i(\varphi^*_{ni}) = f_{Ei},$$

2. The Melitz model: open economy

- As in the closed economy:
  - The cutoffs and market demands are independent of the sector aggregates such as sector spending *X* and labor supply *L*.
  - Only the mass of firms respond to the size of the sectors.

2. The Melitz model: open economy

- Determination of the mass of firms and the price index
  - There are M_{Ei} entrants in a country *i*.
  - A subset M_{ni} $[1 - G_i(\varphi^*_{ni})]\, M_{Ei}$ of these firms export to destination *n*.
  - In country *n* the total mass of sellers is: $M_n = \sum_n M_{ni}.$
  - … and the CES price index is given by:

$$P_n^{1-\sigma} = \sum_i \left\{ M_{ni} \int_{\varphi^*_{ni}}^{\infty} p_{ni}(\varphi)^{1-\sigma} \frac{dG_i(\varphi)}{1 - G_i(\varphi^*_{ni})} \right\} = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \sum_i \left\{ M_{Ei} \tau_{ni}^{1-\sigma} \int_{\varphi^*_{ni}}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) \right\}.$$

  - Besides, we have a relationship between this price index and the demand on market *n*:

$$B_n = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^{\sigma}} X_n P_n^{\sigma-1} \Leftrightarrow P_n^{1-\sigma} = \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \frac{1}{\sigma} X_n B_n^{-1}$$

2. The Melitz model: open economy

Combining the 2 equations above, we can solve out the price index and obtain:

$$\frac{X_n}{\sigma B_n} = \sum_i M_{Ei} \tau_{ni}^{1-\sigma} \int_{\varphi_{ni}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi),$$

This is a system of N equations giving the N entry variables in each country: M_{Ei}

2. The Melitz model: open economy

With: $$B_n = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} X_n P_n^{\sigma-1} \Leftrightarrow P_n^{1-\sigma} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{1}{\sigma} X_n B_n^{-1}$$

and the zero-profit cutoff condition (true for all *n*) :

$$B_n \left(\tau_{ni}\right)^{1-\sigma} \left(\varphi_{ni}^*\right)^{\sigma-1} = f_{ni}$$

We can define the price index in country *n* as a function of the domestic cutoff only (will be useful for welfare analysis):

$$P_n = \frac{\sigma}{\sigma-1} \left(\frac{f_{nn}\sigma}{\beta \bar{L}_n}\right)^{1/(\sigma-1)} \frac{1}{\varphi_{nn}^*}.$$

Many things impact the price index (all cutoffs, prices and transport cost), but the impact of all endogenous is summarized by the domestic entry cuttoff.

2. The Melitz model: open economy

- **The case of a symmetric equilibrium**
- To keep things simple, we consider symmetric trade and production costs across countries

$$\tau_{ni} = \tau \quad \text{and} \quad f_{ni} = f_X \quad \forall n \neq i,$$
$$f_{ii} = f, \quad \text{and} \quad f_{Ei} = f_E \quad \text{and} \quad G_i(\cdot) = G(\cdot) \quad \forall i.$$

2. The Melitz model: open economy

- **The case of a symmetric equilibrium (with N countries)**
  - In this case, the free entry condition yields a common market demand (net of crowding out effect): $B_n = B$ for all countries.
  - This, in turn, implies that all countries have the same domestic cutoff:

$$\varphi_{ii}^* = \varphi^*$$

  - And that there is a single export cutoff: $\varphi_{ni}^* = \varphi_X^*$ for $n \neq i$
  - These cutoffs are the solutions to the zero-profit cutoff conditions:

$$B(\varphi^*)^{\sigma-1} = f, \quad B\tau^{1-\sigma}(\varphi_X^*)^{\sigma-1} = f_X,$$

  - And the free entry condition takes the form:

$$fJ(\varphi^*) + f_X(N-1)J(\varphi_X^*) = f_E.$$

2. The Melitz model: open economy

The case of a symmetric equilibrium

- Zero profit cutoff and free entry condition implies that the two cutoffs (domestic entry and export) are proportional:`

$$\varphi_X^* = \tau \left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*$$

- To observe a selection of firms into export markets ($\varphi_X^* > \varphi^*$), we need to assume: $\tau^{\sigma-1} f_X > f$
- Then we can represent graphically the relationship between productivity and profits

2. The Melitz model: open economy

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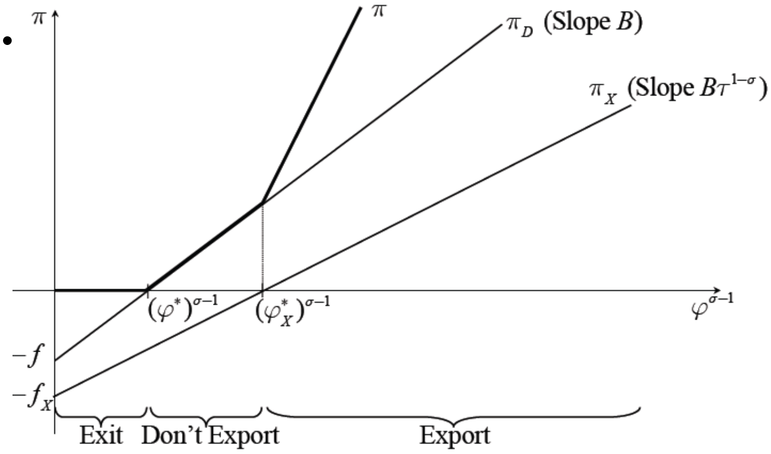
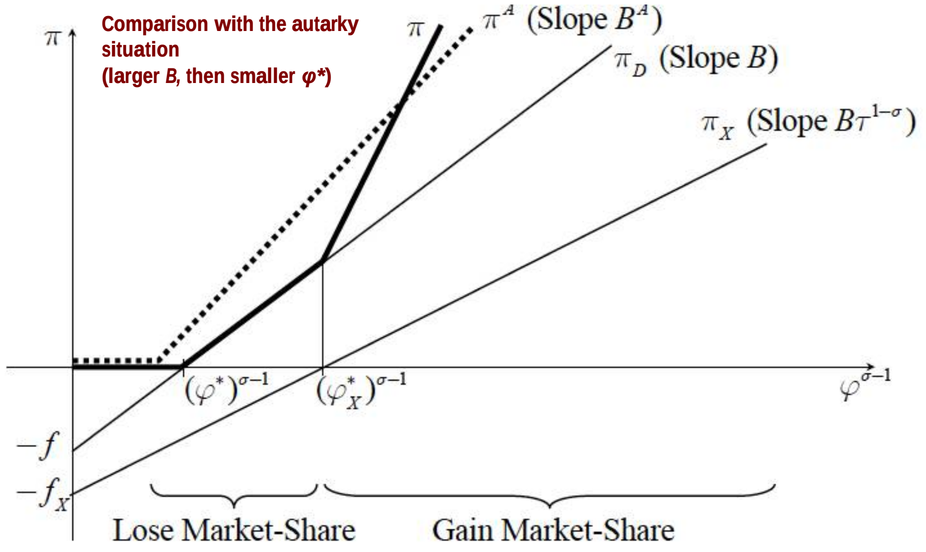


Figure 2: Open Economy Symmetric Countries

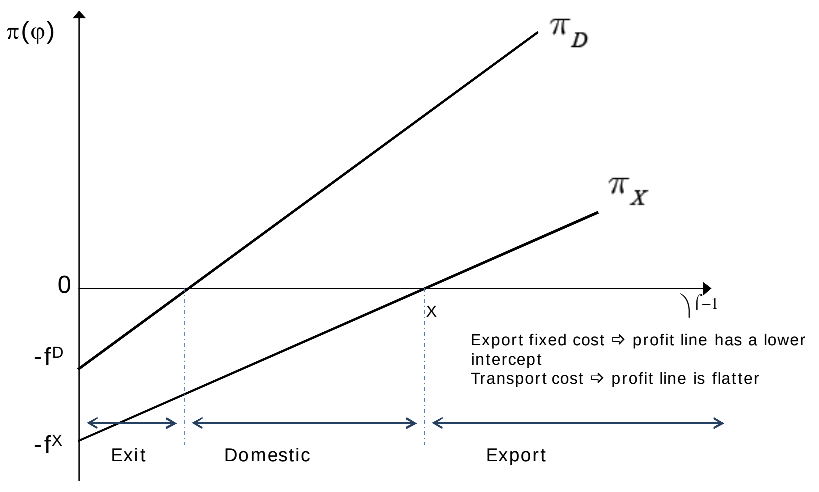
2. The Melitz model: open economy

Comparison with the autarky situation (larger *B*, then smaller *φ)**



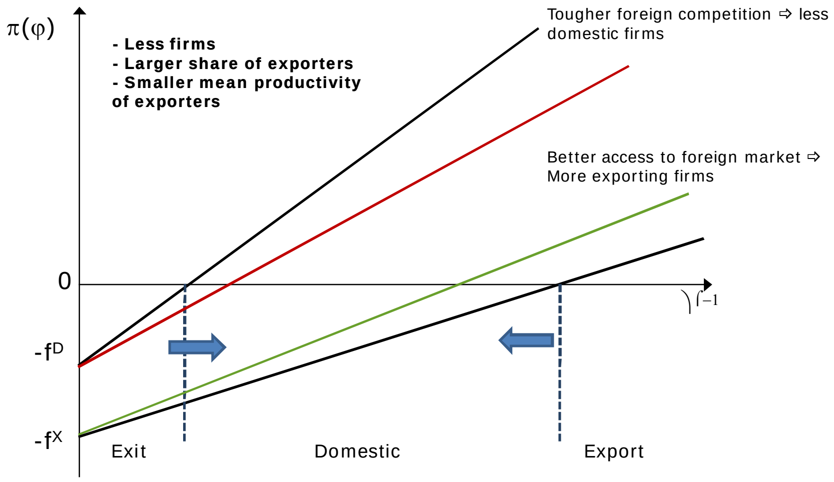
2. The Melitz model: open economy

Let's decompose the effect of trade



2. The Melitz model: open economy

Decrease of transport cost



2. The Melitz model: open economy

Welfare:

Welfare in country *n* is summarized by the price index.

Indeed, with a Cobb-Douglas aggregate utility, welfare per worker in country *n* (with income w_n=*1*) is:

$$\mathbb{U}_n = \prod_{j=0}^{J} P_{nj}^{-\beta_j}$$

Where the price index in country *n* and industry *j* only depends on the domestic cuttoff φ^*_{nnj}

With trade openness, the domestic cutoff increases, the price index decreases and welfare increases.

3. Extension - a: Chaney - 2008

Chaney (AER, 2008):

- Proposes a more simple (but more specific) version of Melitz (2003)
- Details the consequence of trade barriers on the two trade margins:
  - The intensive trade margin = firm-level exports
  - The extensive trade margin = number (or the selection) of exporters
- Shows how firms' heterogeneity changes significantly the predictions of gravity equations

3. Extensions - a: Chaney - 2008

N countries, 2 goods: IRS-MC with heterogeneous firms and an outside good (⇨ wage equalization across countries).

Utility is: $U = q_o^{1-\mu} \left[\int_{\omega \in \Omega} q(\omega)^{\rho} \, d\omega \right]^{\mu/\rho}, \quad 0 < \rho < 1, 0 < \mu < 1,$

We assume a Pareto distribution of productivities

$$G(\varphi) = 1 - \varphi^{-\gamma},$$

γ measures firms' heterogeneity (higher γ = lower variance)

3. Extensions - a: Chaney - 2008

In each country, the mass of entrepreneurs that are allowed to draw a productivity from the distribution *G*, is fixed and proportional to country's size L_j.

Individual revenue of firm *i* on a market *j* is:

Wage

Profit revenues

$$r_{ij}(\varphi) = \left(\frac{\sigma\tau_{ij}}{(\sigma-1)\varphi P_j}\right)^{1-\sigma} \times \mu\left(1+\frac{\Pi}{L}\right)L_j$$

The threshold for this market is:

$$\pi_{ij}(\bar{\varphi}_{ij}) = 0 \Leftrightarrow \bar{\varphi}_{ij} = \lambda_1 f_{ij}^{\frac{1}{\sigma-1}}\left(P_j^{\sigma-1}L_j\right)^{\frac{-1}{\sigma-1}}\tau_{ij}$$

$\lambda_1 = \left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)\left(1+\frac{\Pi}{L}\right)$

3. Extensions - a: Chaney - 2008

- Using the Pareto distribution, one can compute the aggregate bilateral trade:

$$X_{ij} = \lambda \times \frac{L_i L_j}{L} \times \left(\frac{\tau_{ij}}{\theta_j}\right)^{-\gamma} \times f_{ij}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

Exporting country's size

Importing country's size

Distance, etc...

But the coefficient is no longer 1-σ

Multilateral resistance

- This is clearly a gravity equation

3. Extensions - a: Chaney - 2008

- Krugman's gravity equation:

$$\tilde{X}_{ij} = \tilde{\lambda} \times \frac{L_i L_j}{L} \times \left(\frac{\tau_{ij}}{\tilde{\theta}_j}\right)^{-(\sigma-1)}$$

- Chaney's gravity equation:

$$X_{ij} = \lambda \times \frac{L_i L_j}{L} \times \left(\frac{\tau_{ij}}{\theta_j}\right)^{-\gamma} \times f_{ij}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

- The impact of distance and of fixed cost depends on the degree of firms' heterogeneity (and not on price elasticity)

3. Extensions - a: Chaney - 2008

With heterogeneous firms, a change in trade costs affects both the intensive and the extensive margin

Total differentiation gives:

$$\begin{aligned} dX_{ij} = & \left(\int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi)\right) d\tau_{ij} - \left(r(\bar{\varphi}_{ij})\, g(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}}\right) d\tau_{ij} \\ & + \underbrace{\left(\int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial r_{ij}(\varphi)}{\partial f_{ij}} dG(\varphi)\right) df_{ij}}_{\text{Intensive margin}} - \underbrace{\left(r(\bar{\varphi}_{ij})\, g(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial f_{ij}}\right) df_{ij}}_{\text{Extensive margin}} \end{aligned}$$

And

$$\zeta \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \underbrace{(\sigma - 1)}_{\substack{\text{Intensive margin} \\ \text{Elasticity}}} + \underbrace{(\gamma - (\sigma - 1))}_{\substack{\text{Extensive margin} \\ \text{Elasticity}}} = \gamma$$

$$\xi \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}} = \underbrace{0}_{\substack{\text{Intensive margin} \\ \text{Elasticity}}} + \underbrace{\left(\frac{\gamma}{\sigma - 1} - 1\right)}_{\substack{\text{Extensive margin} \\ \text{Elasticity}}} = \frac{\gamma}{\sigma - 1} - 1$$

3. Extensions -b: BRS-2007

Bernard, Redding and Schott (*RES* 2007)

- Plug the Melitz framework into a HO world
- Two-sector, two-country world economy
- Total cost function is:

$$\Gamma_{ij} = \left[f_{ij} + \frac{q_{ij}(\varphi)}{\varphi} \right] (w_{Si})^{\beta_j} (w_{Li})^{1-\beta_j}$$

- ⇨ Trade liberalization increases the zero-profit productivity cutoff in all industries and countries (i.e. more trading firms) and decrease the domestic cuttoff (i.e. low productivity firms wipe out)
- ... but the effect is disproportionately large in comparative advantage industries.

3. Extensions -c: MO-2008

Use a linear demand system (Ottaviano, Tabuchi, and Thisse, 2002)

- Utility is given by:

$$U = q_0^c + \alpha \int_{i\in\Omega} q_i^c \, di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 \, di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i^c \, di \right)^2$$

- Where q_0 is the consumption of a numeraire homogeneous good and q_i is the consumption of a given variety *i*.

3. Extensions -c: MO-2008

$$U = q_0^c + \alpha \int_{i \in \Omega} q_i^c \, di - \frac{1}{2}\gamma \int_{i \in \Omega} (q_i^c)^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega} q_i^c \, di \right)^2$$

- Parameters α, η, and γ are all positive.
- α, η index the substitution pattern between the differentiated varieties and the numeraire: increases in α and decreases in η both shift out the demand for the differentiated varieties relative to the numeraire.
- The parameter γ indexes the degree of product differentiation between the varieties. In the limit when $\gamma = 0$, consumers only care about their consumption level over all varieties (perfect substitute).
- Product differentiation increases with γ

3. Extensions -c: MO-2008

The inverse demand for each variety is then: $p_i = \alpha - \gamma q_i^c - \eta Q^c,$

– Q^c is the overall demand of varieties

And the demand is: $q_i^c = \frac{1}{\gamma}(\alpha - p_i - \eta Q^c)$

⇨ there is a maximum price for wich demand is driven to zero… This threshold depends on the total consumption (i.e. the number of varieties and their price = the competitive pressure on the market).

3. Extensions -c: MO-2008

Total demand is: $Q^c = \frac{N(\alpha - \bar{p})}{\gamma + \eta N}$, where N is the number of consumed varieties and $\bar{p}$ their average price

⇨ total demand for a variety is:

$$q_i \equiv L q_i^c = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_i + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \bar{p}, \qquad \forall i \in \Omega^*,$$

Then, the condition for having a positive production is to be able to set a price lower than:

$$p_i \leq \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta N \bar{p}) \equiv p_{\max}.$$

It decreases when N goes up and $\bar{p}$ goes down, i.e. when competition is fiercer, it is harder to enter the market.

3. Extensions -c: MO-2008

Welfare is given by the indirect utility function:

$$U_i = I_i^c + \frac{1}{2}\left(\eta + \frac{\gamma}{N_i}\right)^{-1} (\alpha - \bar{p}_i)^2 + \frac{1}{2} \frac{N_i}{\gamma} \sigma_{pi}^2$$

I_i^c is the representative consumer's income, $\bar{p}_i$ the average price of varieties, and σ_{pi}^2 is the variance of prices.

Utility increases as the number of active firms increases, their average price decreases and the variance of prices increases.

3. Extensions -c: MO-2008

We assume a fixed cost of entry on the domestic market (f_E). After paying this fixed cost, firm draw their unit labor requirement cost (c), which is the inverse of productivity.

If a firm wants to export, it incurs an iceberg trade cost τ.

In this model, some firms do not export, even in the absence of fixed cost of exporting. Firms drawing a marginal cost such that they charge a price above the threshold price in the domestic market exit immediately.

3. Extensions -c: MO-2008

With:

$$q_i \equiv L q_i^c = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_i + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \bar{p}, \qquad \forall i \in \Omega^*,$$

Profit maximization yields:

$$q(c) = \frac{L}{\gamma}[p(c) - c].$$

3. Extensions -c: MO-2008

Let's call C_D the threshold marginal cost (i.e. the cost of the marginal firm entering the market, i.e. $c_D = p_{max}$)

This cutoff summarizes all domestic market conditions.

We have the following equilibrium firm-level relationships

$p(c) = \frac{1}{2}(c_D + c)$	prices
$\mu(c) = p(c) - c = \frac{1}{2}(c_D - c)$	markups
$r(c) = \frac{L}{4\gamma}\left[(c_D)^2 - c^2\right]$	revenues
$\pi(c) = \frac{L}{4\gamma}(c_D - c)^2$	profits

3. Extensions -c: MO-2008

$p(c) = \frac{1}{2}(c_D + c)$	prices
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$\pi(c) = \frac{L}{4\gamma}(c_D - c)^2$	profits

- Lower cost firms set lower prices and earn higher revenues and profits than firms with higher costs
- Contrary to a CES framework, lower cost firms do not pass on all of the cost differential to consumers in the form of lower prices: **they also set higher mark-ups**

3. Extensions -c: MO-2008

Free entry condition:

- As in Melitz, the expected profit should be equal the sunk cost.

$$\int_0^{c_D} \pi(c)dG(c) = \frac{L}{4\gamma}\int_0^{c_D}(c_D - c)^2 dG(c) = f_E$$

- This defines C_D.
- C_D is also given by the zero profit cut-off function:

$$c_D = \frac{1}{\eta N + \gamma}(\gamma\alpha + \eta N \bar{p})$$

- Which gives the number of firms: $N = \frac{2\gamma}{\eta}\frac{\alpha - c_D}{c_D - \bar{c}}$

3. Extensions -c: MO-2008

- The average cost is unknown. To go further, one needs to assume an explicit distribution of productivities (in order to solve the previous equation and obtain C_D)
- Assuming that productivities (*1/c*) are Pareto distributed with a lower bound $1/c_M$ and a shape parameter *k*, we have the following entry cutoff:

$$c_D = \left[\frac{2(k+1)(k+2)\gamma (c_M)^k f_E}{L}\right]^{1/(k+2)}$$

3. Extensions -c: MO-2008

- Then, the number of active firms is

$$N = \frac{2(k+1)\gamma}{\eta}\frac{\alpha - c_D}{c_D}$$

- And average performances of firms are

$$\bar{c} = \frac{k}{k+1}c_D, \qquad \bar{q} = \frac{L}{2\gamma}\frac{1}{k+1}c_D = \frac{(k+2)(c_M)^k}{(c_D)^{k+1}}f_E,$$
$$\bar{p} = \frac{2k+1}{2k+2}c_D, \qquad \bar{r} = \frac{L}{2\gamma}\frac{1}{k+2}(c_D)^2 = \frac{(k+1)(c_M)^k}{(c_D)^k}f_E,$$
$$\bar{\mu} = \frac{1}{2}\frac{1}{k+1}c_D, \qquad \bar{\pi} = f_E\frac{(c_M)^k}{(c_D)^k}.$$

3. Extensions -c: MO-2008

- In the closed economy, the threshold c_D is lower (and thus average productivity is higher) when:
  - L is higher (the market is bigger)
  - γ is lower (Varieties are closer substitutes)
  - f_E is lower (lower sunk costs)
  - When the toughness of competition is higher, c_D is lower…
  - … and mark-ups are smaller
  - **We have endogenously variable and firm-specific mark-ups**

3. Extensions -c: MO-2008

In this model, market size differences affect productivity cutoff:

- Larger market attract more firms
- this implies fiercer competition in the presence of trade costs
- Then, we have lower zero profit cutoff and higher average productivity on bigger markets.

3. Extensions -c: MO-2008

Trade equilibrium

- Consider 2 countries, *H* and *F* with possible different size.
- Trade incur a per unit cost, such that the cost for delivering a foreign market *l* (*l*= *H, F*) for a firm with a marginal cost *c* is $c\tau^l > c$.
- The maximum price to be active on a market is given by the average price of active firms (including foreigners).

$$p^l_{\max} = \frac{1}{\eta N^l + \gamma}(\gamma \alpha + \eta N^l \bar{p}^l), \quad l = H, F,$$

3. Extensions -c: MO-2008

Trade equilibrium

- As for the closed economy, optimal prices and output levels for a firm with cost *c* can be written as functions of the cuttoff on each markets:

$$p_D^l(c) = \frac{1}{2}(c_D^l + c), \quad q_D^l(c) = \frac{L^l}{2\gamma}(c_D^l - c),$$

$$p_X^l(c) = \frac{\tau^h}{2}(c_X^l + c), \quad q_X^l(c) = \frac{L^h}{2\gamma}\tau^h(c_X^l - c),$$

- Which yields the profits functions:

$$\pi_D^l(c) = \frac{L^l}{4\gamma}(c_D^l - c)^2,$$

$$\pi_X^l(c) = \frac{L^h}{4\gamma}(\tau^h)^2(c_X^l - c)^2.$$

3. Extensions -c: MO-2008

Trade equilibrium

- Free entry condition now contains the domestic and export profits

$$\int_0^{c_D^l} \pi_D^l(c)dG(c) + \int_0^{c_X^l} \pi_X^l(c)dG(c) = f_E.$$

- With a Pareto distribution, we have

$$L^l(c_D^l)^{k+2} + L^h(\tau^h)^2(c_X^l)^{k+2} = \gamma\phi,$$

$$\text{where } \phi \equiv 2(k+1)(k+2)(c_M)^k f_E$$

3. Extensions -c: MO-2008

Trade equilibrium

- The free entry condition can be rewritten:

$$L^l (c_D^l)^{k+2} + L^h \rho^h (c_D^h)^{k+2} = \gamma \phi,$$

where $\rho^l \equiv (\tau^l)^{-k} \in (0,1)$

- ρ is an inverse mesure of trade cost (= a measure of trade openness)

3. Extensions -c: MO-2008

Trade equilibrium

- We thus have the cutoff in a trading economy

$$c_D^l = \left[\frac{\gamma \phi}{L^l} \frac{1-\rho^h}{1-\rho^l \rho^h} \right]^{1/(k+2)}$$

- When entry on foreign market is easier (ρ^h decreases) the cutoff increases (ρ^h<1).
- When entry on domestic market is easier (ρ^l decreases) the cutoff increases.

- As in the closed economy (and with the same equations), this cutoff determines all aggregates and individual behavior and performances.

3. Extensions -c: MO-2008

Trade equilibrium

- With $\rho \in [0,1]$, $\frac{1-\rho^{l}}{1-\rho^{l}\rho^{h}} < 1$
- In a trading economy, the cutoff is lower (more competition induce stricter selection)
- Then:
  - Least productive firms shut down
  - Individual prices and markup decrease = pro-competitive effect
  - Average price decrease (pro-competitive effect + selection)
  - Number of available varieties increase